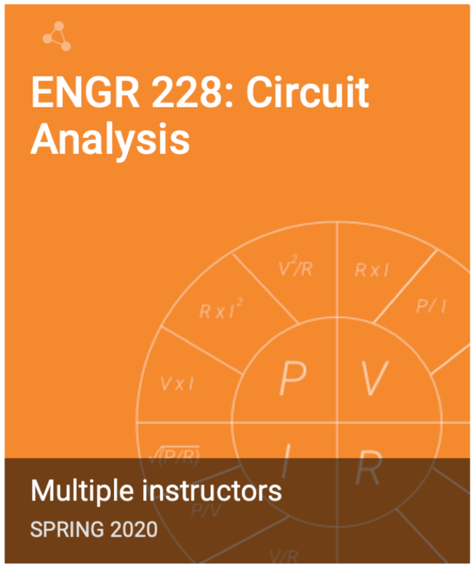


Chapter 7.10

Circuit Analysis Using Phasors

Engr228 - Circuit Analysis

Spring 2020

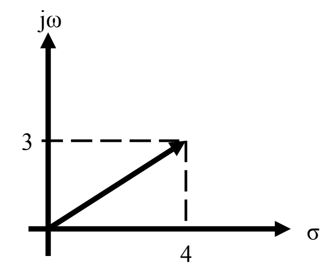
Dr Curtis Nelson

Section 7.10 Objectives

- Be able to transform a circuit with sinusoidal sources into the frequency domain using phasor concepts;
- Know how to use the following circuit analysis techniques to solve a circuit in the frequency domain:
  - Ohm’s Law;
  - Kirchhoff’s laws;
  - Series and parallel simplifications;
  - Voltage and current division;
  - Node-voltage method;
  - Mesh-current method;
  - Thévenin and Norton equivalents;

Review of Complex Numbers

- Complex numbers can be viewed as vectors where the X-axis represents the real part and the Y-axis represents the imaginary part.
- There are two common ways to represent complex numbers:
  - Rectangular form: $4 + j3$
  - Polar form: $5 \angle 37^{o}$

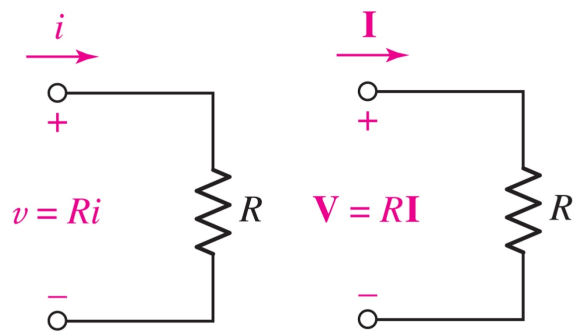


Phasors

- A ***phasor*** is a vector that represents an AC electrical quantity such as a voltage waveform or a current waveform;
- The phasor's length represents the peak value of the voltage or current;
- The phasor's angle represents the phase angle of the voltage or current;
- Phasors are used to represent the relationship between two or more waveforms with the same frequency.

Phasors: The Resistor

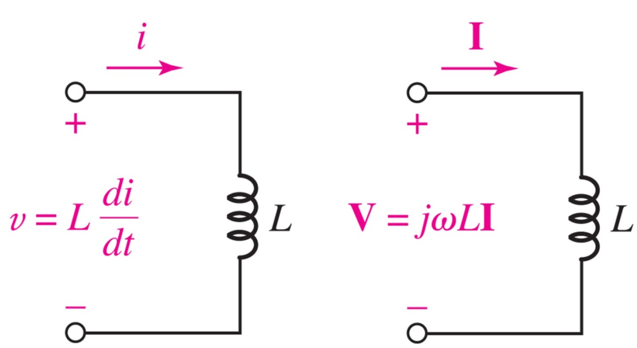
In the ***Frequency Domain***, Ohm's Law takes the same form:



Phasors: The Inductor

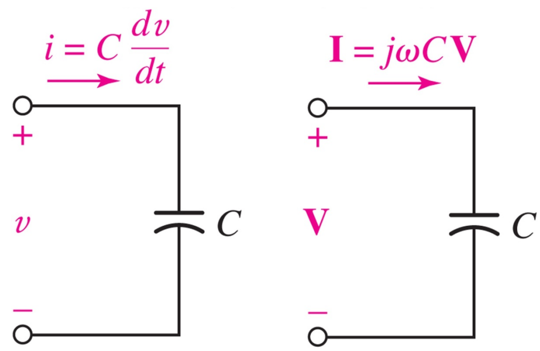
By dividing the phasor voltage by the phasor current, we derive an expression for the phasor impedance of an inductor shown in the figure below.

Differentiation in time becomes multiplication in phasor form: (calculus becomes algebra).



Phasors: The Capacitor

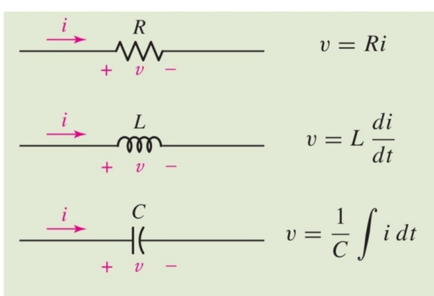
Differentiation in time becomes multiplication in phasor form:
(calculus becomes algebra again).

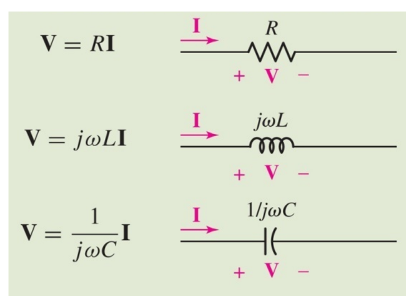


Summary: Phasor Voltage/Current Relationships

Time Domain

Frequency Domain



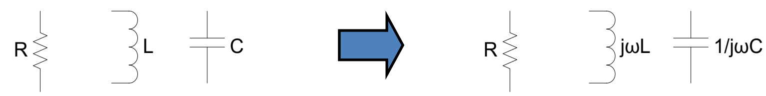


Calculus (real numbers)

Algebra (complex numbers)

Circuit Analysis Procedure Using Phasors

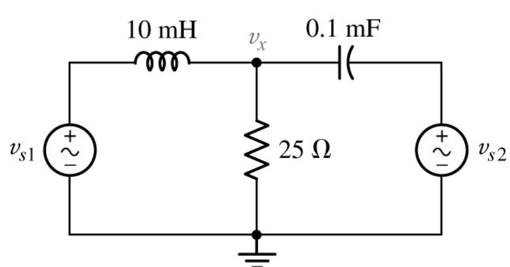
- Change the voltage/current sources into phasor form;
- Change R, L, and C values into phasor impedances;



- Use normal DC circuit analysis techniques but the values of voltage, current, and impedance can be complex numbers;
- Change back to the time-domain form if required.

Example Problem

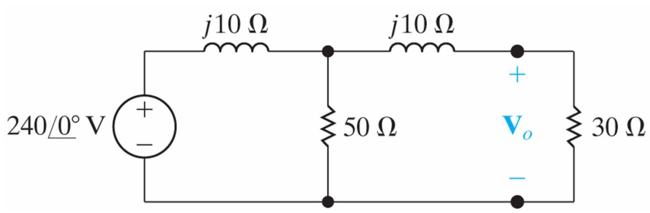
Find v_x(t) in the circuit below if v_{s1} = 20cos1000t V and v_{s2} = 20sin1000t V.



$v_x(t) = 70.71\cos(1000t - 45^o)\ V$

Example Problem 9.55 (Nilsson 11th)

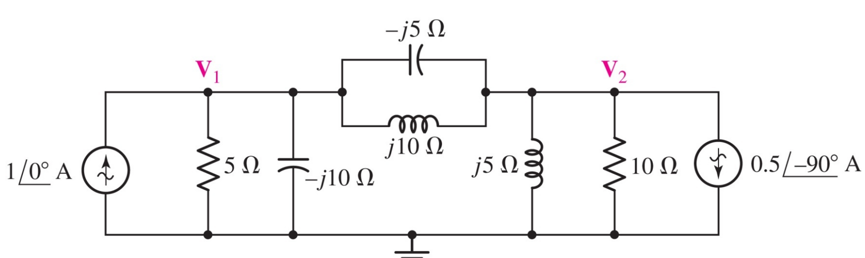
Use the node-voltage method to find $\mathbf{V}_o$.



Answer: $V_O = 138.078 - j128.22V = 188.43\angle{-42.88^o}\ V$

Nodal Analysis Example

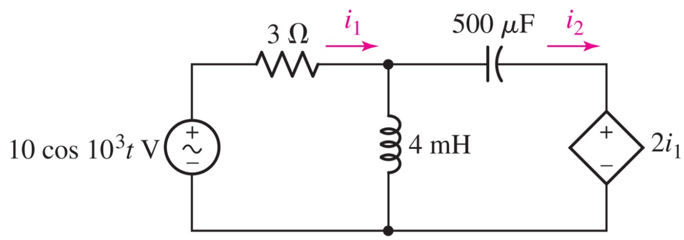
Find the phasor voltages V_1 and V_2.



Answer: $V_1 = 1 - j2$ V *and* $V_2 = -2 + j4$ V

Mesh Analysis Example

Find the currents $i_1(t)$ and $i_2(t)$.

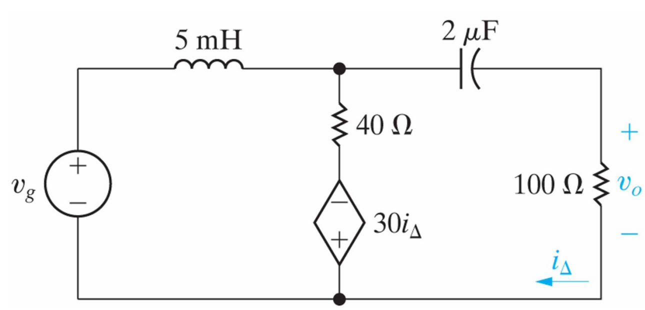


$i_1(t)$ = $1.24\cos(10^3t + 29.7^\circ)$ A

$i_2(t)$ = $2.77\cos(10^3t + 56.3^\circ)$ A

Example Problem 9.64 (Nilsson 9th)

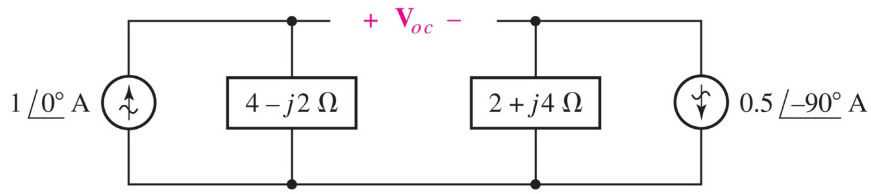
Use the mesh current method to find the steady-state expression for v_o if v_g = 130cos(10,000t)V.



Answer: $v_o = 56.57\cos(10{,}000t - 45^\circ)V$

Thévenin Example

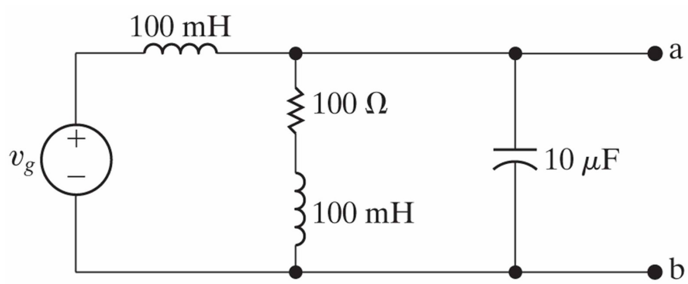
Thévenin's theorem also applies to phasors; use it to find $\boldsymbol{V_{OC}}$ and $\boldsymbol{Z_{TH}}$ in the circuit below.



Answer: $V_{oc} = 6 - j3\ V$ $\quad Z_{TH} = 6 + j2\ \Omega$

Example Problem 9.44 (Nilsson 9th)

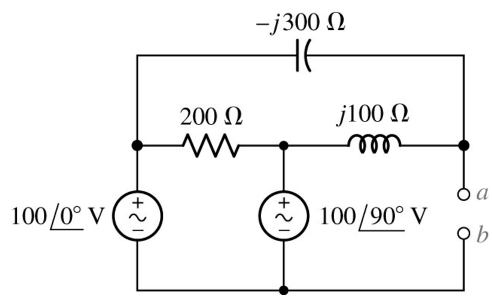
Find the Thévenin equivalent circuit at terminals ab for $v_g = 247.49\cos(1000t + 45°)$ V.



$V_{TH} = 350V = 350\angle 0^o\ V$
$Z_{TH} = 100 + j100\Omega = 141.4\angle 45^o\ \Omega$

Example Problem

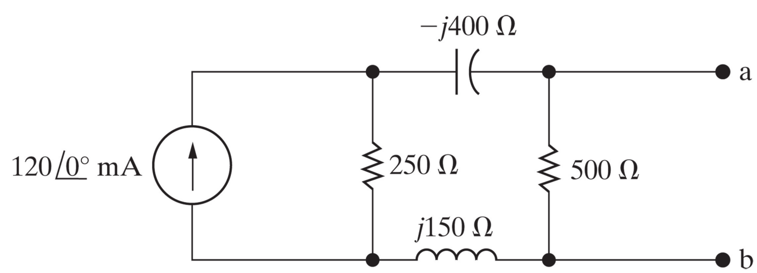
Find the Thévenin equivalent circuit at terminals *ab*.



$V_{TH} = -50 + j150 = 158.11\angle 108.43^o\ V$
$Z_{TH} = j150\Omega$

Example Problem 9.45 (Nilsson 10th)

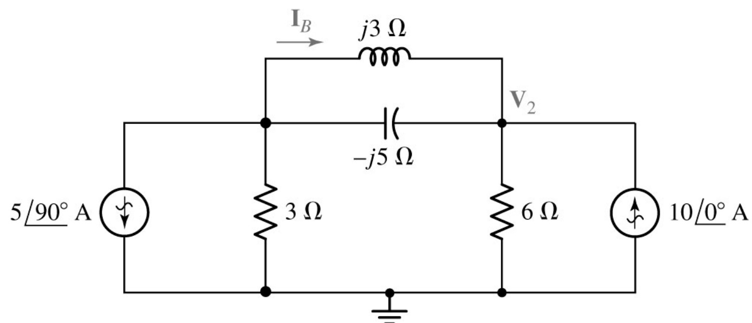
Use source transformations to find the Thévenin equivalent circuit with respect to terminals a and b.



$V_{TH} = 18 + j6\ V,\quad R_{TH} = 200 - j100\Omega$
$V_{TH} = 18.97 \angle 18.43^o\ V,\quad R_{TH} = 223.6 \angle -26.56^o\Omega$

Example Problem

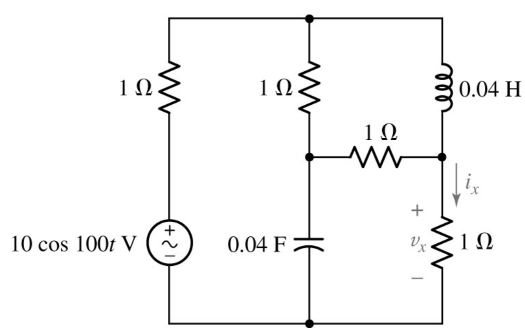
Find v_2(t).



$v_2(t) = 34.36\cos(\omega t + 23.63°)V$

Example Problem

Find v_X(t).



$v_X(t) = 1.213\cos(100t - 75.96°)V$

Section 7_10 Summary

From the study of this section, you should:

- Understand phasor concepts;
- Be able to transform a circuit with sinusoidal sources into the frequency domain using phasor concepts;
- Know how to use the following circuit analysis techniques to solve a circuit in the frequency domain:
  - Ohm's Law;
  - Kirchhoff's laws;
  - Series and parallel simplifications;
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  - Node-voltage method;
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